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Transient conjugated heat transfer in thick walled pipes with convective boundary conditions

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Abstract

Transient conjugated heat transfer for laminar flow in the thermal entrance region of pipes is investigated by considering two dimensional wall and axial fluid conduction. The problem is handled for an initially isothermal, infinitely long, thick-walled and two-regional pipe for which the upstream region is insulated and solved numerically by a finite difference method for hydrodynamically developed flow with a step change in the ambient fluid temperature in the heated downstream region. A parametric study is done to analyse the effects of five defining parameters namely, wall thickness ratio, wall-to-fluid conductivity ratio, wall-to-fluid thermal diffusivity ratio, the Peclet number and the Biot number.

1. Introduction

Analysis of unsteady conjugated heat transfer is important for the control of heat exchangers during start up, shutdown or any change in the operating conditions. To understand the transient behaviour of heat transfer processes in devices for which the operating conditions are changing periodically or accidentally, due to a failure of the power source or pumps of the system, is also important to prevent the undesirable reduce in thermal performance and thermal stresses which may produce mechanical failure. Transient heat transfer in laminar pipe or channel flow has been studied by many investigators with step or periodic change in either the boundary or fluid inlet conditions. Some of these investigators have considered extremely thin walled conduits for which the effects of wall conduction may be ignored and the conditions for the outer surface are assumed to prevail along the inner surface. In conjugated problems however, the conditions for wall–fluid interface are not known a priori and therefore, the energy equations in both solid and fluid sides should be solved simultaneously by the conditions of continuity in temperatures and in heat fluxes for the interface. In some of these problems the radial wall conduction is neglected while in some works both radial and axial wall conduction is considered. An outline of steady conjugate problems in literature is given in [1]. Wijeysundera [2] analysed a steady conjugated problem with convective boundary conditions for pipes and rectangular channels heated in a finite region, by considering the wall conduction in the axial direction.

Many problems were investigated for one-regional pipes or channels by neglecting the axial fluid conduction. However when flow Peclet number is low, the axial conduction in the fluid may be comparable to convection in the thermal entrance region and the diffusion of heat backward through the upstream side may cause the temperature profiles to develop before the beginning of the heating section. Therefore such problems should be analysed for two-regional pipes and heat transfer characteristics should be determined for both the upstream and the downstream regions. A literature survey on the effect of axial fluid conduction in pipe or channel flows is given in [3]. An early investigation analysing the effects of axial fluid conduction with convective boundary conditions at the outer surface was realised by Shneider [4] and the problem was solved for slug flows in pipes or

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Nomenclature

a	constant of discretization equation (Eqs. (6a)–(6j))
Bi	Biot number
c_p	specific heat at constant pressure
d	pipe wall thickness
Fo	Fourier number
Gz	Graetz number
h	convective heat transfer coefficient
k	thermal conductivity
Nu	Nusselt number
Pe	Peclet number
q	heat flux
r	radial coordinate
t	time
T	temperature
T_0	initial temperature of the system
T_1	ambient temperature
u	axial velocity
x	axial coordinate

Greek symbols

α	thermal diffusivity
δr	radial position difference
δx	axial position difference
Δr	radial step size
Δt	time step increment
Δx	axial step size
ρ	density

Subscripts

b	bulk
f	fluid
i	inner wall
i, j	at nodal point i, j
m	mean
o	outer wall
w	wall
wf	ratio of wall to fluid

Superscripts

$'$	dimensionless quantity
0	at previous time step

channels. Similar problems were solved in pipes for slug flow by Vick and Özisik [5] and with parabolic velocity profile by Vick et al. [6] and by Lee and Hwang [7] and with both convective and radiative boundary conditions by Campo and Auguste [8]. Olek [9] studied an analytical problem in duct flow of non-Newtonian fluids with axial conduction and convecting heat from the duct surface. Steady conjugated problems considering both the wall conduction and the axial fluid conduction are very limited and an outline is also given in [1]. Unconjugated heat transfer in the thermal entrance region of pipes with convective boundary conditions was worked by Sucec [10].

The first investigator worked for transient conjugated heat transfer was also Sucec [11] and he analysed the problem for slug flow in one-regional parallel plates with variable inlet fluid or wall temperatures. Krishan [12] has worked for hydrodynamically developed pipe flow under step change in either outside wall temperature or heat flux and, Sucec and Sawant [13] for parallel plates with periodically changing inlet temperature. Problems were solved by Sucec [14,15] for one regional parallel plates with step change in ambient fluid temperature, numerically by a finite difference method and analytically by Laplace transformation technique. In both of his works he investigated the problem for a sudden change in the ambient fluid temperature and compared the results with quasi-steady solutions. He indicated that the quasi-steady solutions have appreciable error especially in the early transient periods. Cotta et al. [16] analysed the problem with slug flow and periodically changing inlet fluid temperature for parallel plates and circular pipes. Lin and Kuo [17] worked with a step change at the outer wall heat flux and found that wall-to-fluid heat capacity ratio, among other parameters, has the decisive impact on the unsteady heat transfer in the flow. Yan et al. [18], in their work with a step change at the outer surface temperature for pipes, indicated that the ratios of wall-to-fluid thermal diffusivity, thermal conductivity and outside and inside radii have pronounced effects on unsteady heat transfer on laminar pipe flows and it is necessary to consider wall effects in many analysis. Travelho and Santos [19] worked with slug flow and varying inlet temperature for parallel plates and Olek et al. [20] with parabolic velocity profiles for pipes. They concluded that the degree of conjugation and viscous dissipation have a great impact on the temperature distribution in the fluid.

Some numerical solutions were developed considering two-dimensional wall conduction for thick walled pipes. Shutte et al. [21] analysed the problem for flows in pipes and for simultaneous transient development with a step change in constant outside wall heat flux. They considered two cases, either for transient heat transfer in steady developing flow or for simultaneous development of flow and heating. Their results emphasised the influence of parameters on the duration of transient and that all have significant influence. Lee and Yan [22] worked with a step change in constant outside surface temperature and showed that the unsteady heat transfer characteristics are strongly dependent on problem parameters. Yan [23] considered a step change in the ambient temperature and by including the axial fluid conduction in channels heated in a finite region and found that particularly the wall-to-fluid thermal diffusivity ratio plays a significant role on the speed of propagation of thermal energy in the fluid region. Li and Kakaç [24] developed a theoretical solution for channel flow with a step change either in constant ambient temperature or constant wall heat flux, accompanying with sinusoidally changing inlet fluid temperature. Their results show that, the external convection, or Biot number on the temperature responses along the duct is only important for high wall-to-fluid thermal capacitance ratios. Recently, Bilir [25] in a similar work analysed the problem with a step change in the constant outside surface temperature in an infinite two-regional pipe. He concluded that for problems in which axial fluid conduction is important, the thermal inertia of the system is mainly dependent on the flow conditions rather than to be on the wall characteristics. Sucec and Weng [26] worked for parallel plates analytically with linearly changing wall heat flux, Sucec [27] numerically with sinusoidal wall heat generation and Kiwan and Al-Nimr [28] analytically for semi-infinite pipes and ducts for slug flow. Yan [29] analysed the problem for hydrodynamically developed turbulent flows in pipes with the k–ε model.

2. Problem formulation

In the present work a transient conjugated heat transfer problem is analysed in thick walled pipes for thermally developing laminar flows. The effects of two-dimensional wall conduction and the axial fluid conduction for low Peclet number flows were considered with a convective boundary condition. The schematics of the problem and the coordinate system are shown in Fig. 1.

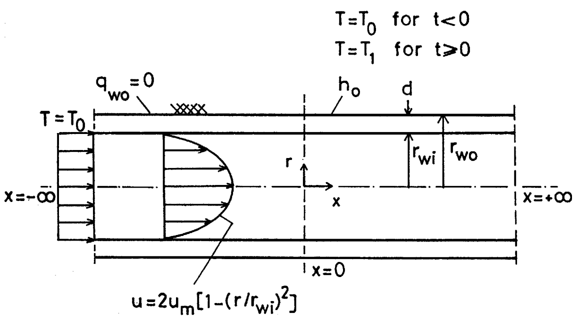

Fig. 1. Schematics of the problem and coordinate system.

The flow pipe is two-regional and extends infinitely in both directions. Far in the upstream side ($x = -\infty$) the fluid temperature is T_0 and uniform. The upstream side of the wall is externally insulated and at the beginning of the heated downstream region the flow is hydrodynamically developed. Initially the whole system is isothermal at temperature T_0 and at time $t = 0$ the ambient temperature suddenly raises to a new value T_1 and remains constant until the system attains the steady state. The pipe receives heat by convection from the ambient fluid with a heat transfer coefficient h_0, which is constant along the outer wall surface in the downstream region. Physical properties of the fluid are assumed to be constant and viscous dissipation is neglected.

The above-described problem may be formulated in non-dimensional form as follows. In the wall side, the differential equation is

$$\frac{1}{\alpha_{\rm wf}}\frac{\partial T'_{\rm w}}{\partial t'} = \frac{1}{r'}\frac{\partial}{\partial r'}\left(r'\frac{\partial T'_{\rm w}}{\partial r'}\right) + \frac{1}{Pe^2}\frac{\partial^2 T'_{\rm w}}{\partial x'^2}. \tag{1a}$$

The initial and boundary conditions are

$$\text{at } t' = 0 \quad T'_{\rm w} = 0; \tag{1b}$$

$$\text{at } x' = -\infty \quad T'_{\rm w} = 0; \tag{1c}$$

$$\text{at } x' = +\infty \quad \frac{\partial T'_{\rm w}}{\partial x'} = 0 \quad (T'_{\rm w} = 1 \text{ at steady state}); \tag{1d}$$

$$\text{at } r' = 1 + d' \quad \text{for } x' < 0 \quad \frac{\partial T'_{\rm w}}{\partial r'} = 0; \tag{1e}$$

$$\text{at } r' = 1 + d' \quad \text{for } x' \geqslant 0 \quad \frac{\partial T'_{\rm w}}{\partial r'} + Bi(T'_{\rm w} - 1) = 0; \tag{1f}$$

$$\text{at } r' = 1 \quad T'_{\rm w} = T'_{\rm f}; \tag{1g}$$

$$\text{at } r' = 1 \quad \frac{\partial T'_{\rm w}}{\partial r'} = \frac{1}{k_{\rm wf}}\frac{\partial T'_{\rm f}}{\partial r'}. \tag{1h}$$

In the fluid side, the differential equation is

$$\frac{\partial T'_{\rm f}}{\partial t'} + (1 - r'^2)\frac{\partial T'_{\rm f}}{\partial x'} = \frac{1}{r'}\frac{\partial}{\partial r'}\left(r'\frac{\partial T'_{\rm f}}{\partial r'}\right) + \frac{1}{Pe^2}\frac{\partial^2 T'_{\rm f}}{\partial x'^2}. \tag{2a}$$

The initial and boundary conditions are

$$\text{at } t' = 0 \quad T'_{\rm f} = 0; \tag{2b}$$

$$\text{at } x' = -\infty \quad T'_{\rm f} = 0; \tag{2c}$$

$$\text{at } x' = +\infty \quad \frac{\partial T'_{\rm f}}{\partial x'} = 0 \quad (T'_{\rm f} = 1 \text{ at steady state}); \tag{2d}$$

$$\text{at } r' = 1 \quad T'_{\rm f} = T'_{\rm w}; \tag{2e}$$

$$\text{at } r' = 1 \quad \frac{\partial T'_{\rm f}}{\partial r'} = k_{\rm wf}\frac{\partial T'_{\rm w}}{\partial r'}; \tag{2f}$$

$$\text{at } r' = 0 \quad \frac{\partial T'_{\rm f}}{\partial r'} = 0. \tag{2g}$$

Non-dimensional parameters of the problem are defined as:

$$T' = \frac{T - T_0}{T_1 - T_0}, \quad x' \frac{x}{r_{\mathrm{wi}} Pe} \equiv \frac{2}{Gz}, \quad r' = \frac{r}{r_{\mathrm{wi}}},$$
$$d' = \frac{d}{r_{\mathrm{wi}}}, \quad k_{\mathrm{wf}} = \frac{k_{\mathrm{w}}}{k_{\mathrm{f}}}, \quad \alpha_{\mathrm{wf}} = \frac{\alpha_{\mathrm{w}}}{\alpha_{\mathrm{f}}}, \quad t' = \frac{t\alpha_{\mathrm{f}}}{r_{\mathrm{wi}}^2} \equiv Fo,$$
$$Pe = \frac{2u_{\mathrm{m}} r_{\mathrm{wi}} \rho_{\mathrm{f}} c_{\mathrm{pf}}}{k_{\mathrm{f}}} \quad \text{and} \quad Bi = \frac{h_{\mathrm{o}} r_{\mathrm{wi}}}{k_{\mathrm{w}}}.$$

Dimensionless fluid bulk temperatures, T'_{b} interfacial heat flux values, q'_{wi}, and local Nusselt numbers, Nu, may be of engineering interest and can be calculated as follows:

$$T'_{\mathrm{b}} = 4 \int_0^1 r'(1 - r'^2) T'_{\mathrm{f}} \, \mathrm{d}r', \tag{3}$$

$$q'_{\mathrm{wi}} = -\left(\frac{\partial T'_{\mathrm{f}}}{\partial r'} \right)_{r'=1}, \tag{4}$$

$$Nu = \frac{2q'_{\mathrm{wi}}}{T'_{\mathrm{wi}} - T'_{\mathrm{b}}}. \tag{5}$$

3. Solution methodology

The systems of Eqs. (1a)–(1h) and (2a)–(2g) are solved simultaneously by a numerical finite-difference method. The conductive terms in Eqs. (1a) and (2a) are discretized by central-difference schemes and the convective terms in Eq. (2a) by an exact method which is given in [3]. This method of discretization may be treated as an application, for two-dimensional cylindrical systems, of the general method defined as "exact or exponential scheme" by Patankar [30]. For the transient terms in the equations a fully implicit formulation is applied to assure stability in the solutions. Here omitting the details of the deriving procedure, the following discretization equation is obtained for an interior (non-boundary) nodal point (i, j) both in the wall and in the fluid side:

$$a_{i,j} T'_{i,j} = a_{i+1,j} T'_{i+1,j} + a_{i-1,j} T'_{i-1,j} + a_{i,j+1} T'_{i,j+1} + a_{i,j-1} T'_{i,j-1} + a^{\mathrm{o}}_{i,j} T'^{\mathrm{o}}_{i,j}, \tag{6a}$$

where in the fluid side

$$a_{i+1,j} = \frac{(r'_j - r'^3_j)(\Delta r')_j}{\exp[Pe^2(1 - r'^2_j)(\delta x')_{i+1}] - 1}, \tag{6b}$$

$$a_{i-1,j} = \frac{(r'_j - r'^3_j)\exp[Pe^2(1 - r'^2_j)(\delta x')_{i-1}](\Delta r')_j}{\exp[Pe^2(1 - r'^2_j)(\delta x')_{i-1}] - 1}, \tag{6c}$$

$$a^{\mathrm{o}}_{i,j} = \frac{r'_j(\Delta x')_i(\Delta r')_j}{\Delta t'}, \tag{6d}$$

in the wall side

$$a_{i+1,j} = \frac{r'_j(\Delta r')_j}{Pe^2(\delta x')_{i+1}}, \tag{6e}$$

$$a_{i-1,j} = \frac{r'_j(\Delta r')_j}{Pe^2(\delta x')_{i-1}}, \tag{6f}$$

$$a^{\mathrm{o}}_{i,j} = \frac{r'_j(\Delta x')_i(\Delta r')_j}{\alpha_{\mathrm{wf}} \Delta t'}, \tag{6g}$$

and in both sides

$$a_{i,j+1} = \frac{r'_{j+1}(\Delta x')_i}{(\delta r')_{j+1}}, \tag{6h}$$

$$a_{i,j-1} = \frac{r'_{j-1}(\Delta x')_i}{(\delta r')_{j-1}}, \tag{6i}$$

$$a_{i,j} = a_{i+1,j} + a_{i-1,j} + a_{i,j+1} + a_{i,j-1} + a^{\mathrm{o}}_{i,j}. \tag{6j}$$

The finite-difference scheme used in the discretization of the differential equations and the boundary conditions was also used in some previous works [1,3,25] and proved to give fast and reliable results in heat transfer problems for thermally developing laminar pipe flows, especially when axial fluid conduction is important, i.e. for low Peclet number flows.

Due to axial symmetry, the grid system is bounded between the outer surface and the axis of the pipe, while the boundaries in the axial direction is guessed from the results of a similar work [25] and tested by some coarse grid systems as to satisfy the conditions at that boundaries. The grids are laid both in the wall and in the fluid sides and contracted radially near the interface in both sides. Axial grids are also contracted in the vicinity of the beginning of the heating zone. The first axial step size is taken to be 0.001 for both upstream and downstream regions and linearly stretched in both directions by taking the axial step size of a grid as 1.35 times as the previous grid. Although the number of grids in the axial direction depends on the length of the computational region and therefore on the parameter values, generally 17×58 grid systems are used except for some extreme parameter combinations. A non-uniform time step is also used to speed up the solutions and to ensure accuracy. The first time step is taken to be 0.0001 and increased by 10% in the subsequent steps.

Temperature distributions are obtained by Gauss–Seidel iteration technique. At each time step temperatures are found by the line-by-line method [30] by traversing the grid points from outer surface of the pipe wall to axis and by sweeping from upstream to downstream. The harmonic mean approach of Patankar [30] is used to descretize the boundary conditions at the interface and a consecutive procedure is used in the solutions. In iterations, Eq. (2e) and therefore the previously calculated interfacial temperatures is used as a boundary condition for the fluid side. When the iteration is continued in the wall side, Eq. (1h) and therefore interfacial heat flux values is used to transfer information from the fluid to the wall side.

Iterations were continued until convergence up to the fourth decimal point and at the time step when the number of iterations decreased to be less than two, the system was assumed to reach the steady state. It is observed that the convergence was quite rapid and at the beginning of transient, the solutions were obtained in a few numbers of iterations since small time steps are chosen for that period. At succeeding times the number of iterations increased and when the system approaches to the steady state it again decreased sharply. Although the number of iterations and time steps depend on the parameter values, generally the results were obtained in 30–100 iterations at each time step and around a total number of 4000 iterations for a run. The method used was controlled by some accuracy tests, as by changing the number and the positions of the grid points, traversing and sweeping directions, time steps and convergence limit and differences in computed values for any case were not considerable.

4. Results and discussion

The problem solved depends on five parameters namely, wall thickness ratio, d', wall-to-fluid thermal conductivity ratio, k_{wf} wall-to-fluid thermal diffusivity ratio, α_{wf}, the Peclet number, Pe and the Biot number, Bi. Solutions are made for different combinations of these parameters: $d' = 0.02$, 0.1 and 0.3; $k_{wf} = 0.1$, 1, 10, 100 and 1000; $\alpha_{wf} = 0.1$, 1, 10, 100 and 1000; $Pe = 0.5$, 1, 5 and 20 and $Bi = 0.1$, 1, 10, 100, 1000 and 10 000. These values were selected as appropriate for problems of engineering interest and from the range that all the presumed effects of the defined problem, i.e. two-dimensional wall and axial fluid conduction are in a significant level. The results are given in interfacial heat flux values.

In Fig. 2, axial distribution of non-dimensional interfacial heat fluxes are given at several instants of time for a typical combination of some average parameter values. As shown in the figure, in the upstream region, both in the wall and in the fluid side there is substantial amount of heat transfer due to axial conduction. As time elapses, more diffusion of heat backwards causes this preheating length to increase. At early times of transient, the wall axial conduction is more rapid than in the fluid side and therefore the upstream heat flux values increase with time. After the fluid axial conduction becomes more pronounced, temperatures in the fluid side increase while the increase in the wall side temperatures slows down and so the heat flux values decrease. This is the reason why the curves for different times intersect each other. In the downstream region at early times the curves rise and attain a constant value and with the time elapsing, after rising to a maximum a decrease is shown in the axial distribution of heat flux values. At the beginning of the transient, inside wall temperatures increase more rapidly than the fluid temperatures and so heat transfer values increase. As time goes on, increase in fluid temperatures becomes faster than increase in the wall side and heat flux values decrease. On the other hand, by the increasing time, the increase in the convection effect results a decrease in both the peak and the average heat flux values and also disturbs the uniformity of the curves in the downstream region. This trend continues until the system reaches to its final steady state.

Fig. 3 is given to analyse the effect of wall thickness ratio on interfacial heat flux. The curves are drawn for three different instants of time in the transient by parametizing for several values of thickness ratio. In thin walled pipes, heat entering from the outer surface is easily transferred through the interface since the thermal resistance and the capacity of the wall is low. Therefore,

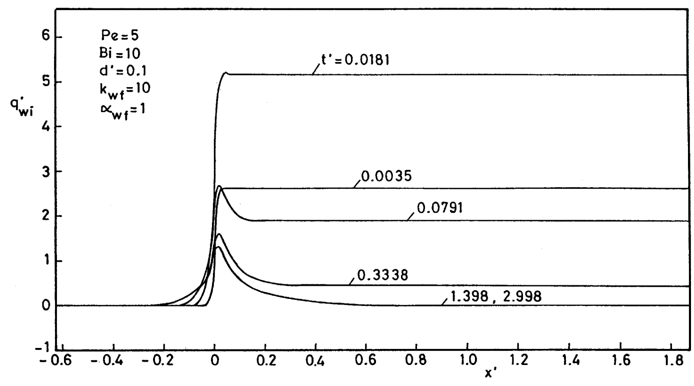

Fig. 2. Transient axial distribution of interfacial heat flux.

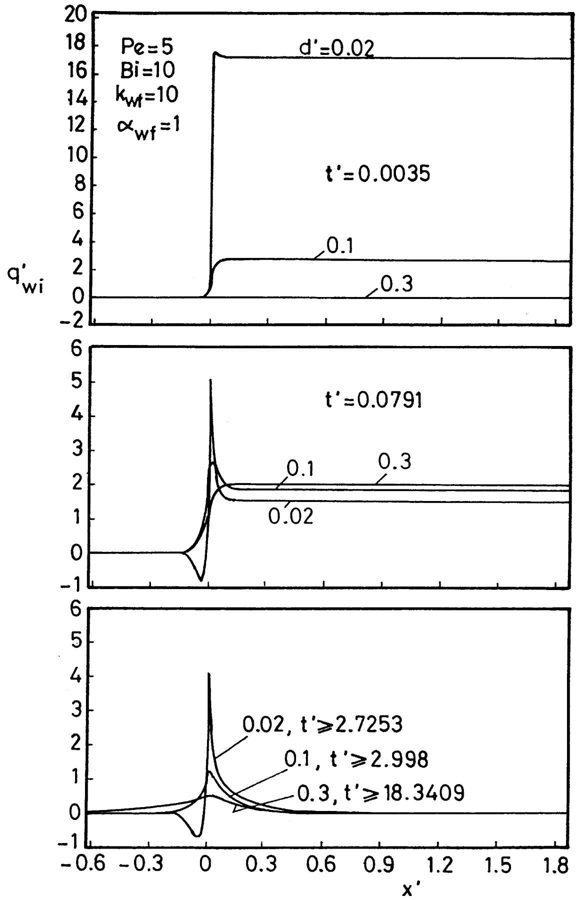

Fig. 3. Effect of wall thickness ratio on interfacial heat flux.

at early times interfacial heat flux values are high. By the same reason, for thin walls the interface temperatures in the downstream region are high and therefore heat flux values are also higher in steady state. However in intermediate transient periods, an opposite situation is shown. With elapsing time the increase in the effect of convection begins earlier in thin walled pipes and for that period heat flux values are lower than those in thick walled pipes, and the curves intersect each other in the downstream region.

An interesting feature arising for thin walled pipes is the negative heat flux values shown in the upstream region. In the early transient, in the vicinity of the end of the upstream region, lower but positive heat flux values are obtained. As time increases the heat penetrated through the upstream region by axial wall conduction is easily transferred to the fluid side since the thermal resistance of the wall in the radial direction is very small. On the other hand, by the axial fluid conduction the diffusion of heat backward through the upstream side in the fluid region results higher bulk temperatures than interface temperatures. Thus, heat transfer from the fluid to the wall side in the upstream region of thin walled pipes causes negative heat flux values. On the other hand, the thermal development lengths in the downstream region are almost the same for thin and thick walls. The increase in wall thickness increases the thermal inertia of the system and therefore the time to reach the steady state is longer for increasing pipe thickness.

In Fig. 4 the effect of wall-to-fluid thermal conductivity ratio on interfacial heat flux is shown. At the beginning of the transient high values of heat flux are obtained since greater k_{wf} decreases the wall resistance in radial direction. On the other hand, with high k_{wf} the extent and the magnitude of heat flux values increase in the upstream region. This can be explained by the diffusion of heat faster and for longer distances in the wall side due to the decreased thermal inertia and increased conductivity for high k_{wf} values. For greater k_{wf} values the rate of heat transfer to the fluid is greater and therefore the thermal development lengths are small. Since high k_{wf} means high α_{wf} in relative sense, and therefore lower thermal inertia, the time to reach the steady state is lower for high k_{wf} values. Another result, which may be deduced from this figure, is that, the effect

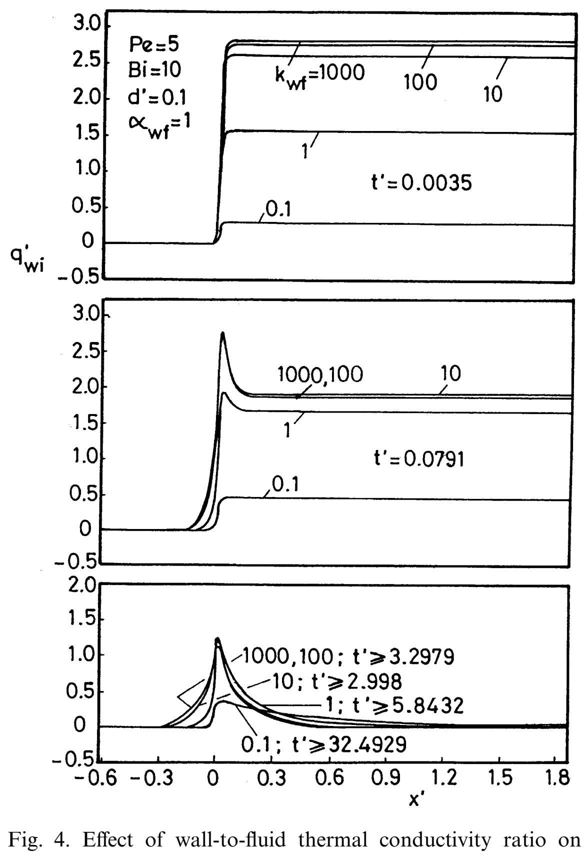

Fig. 4. Effect of wall-to-fluid thermal conductivity ratio on interfacial heat flux.

of k_{wf} decreases as it increases and as the time elapses. It can be seen that the effect of this parameter can be neglected if $k_{wf} > 100$.

The effect of wall-to-fluid thermal diffusivity ratio on interfacial heat flux is given in Fig. 5. At initial times for high α_{wf}, heat flux values are also high due to small thermal capacity of the wall. As time increases, the trend of decrease in heat flux values is more rapid for greater α_{wf}, resulting from increasing convection effect. In the upstream region for high α_{wf} values, the extent and the magnitude of interfacial heat flux values are greater. This may be explained by longer and faster diffusion of heat backwards in the wall side due to the decreased thermal inertia and increased conductivity. Although α_{wf} has considerable influence on heat transfer especially at early and intermediate periods of transient, the time to reach the steady state does not change with this parameter (except, $\alpha_{wf} = 0.1$). Excluding again this value, the final shapes of the curves in steady state are the same as expected. Another distinguishing feature of the curves for $\alpha_{wf} = 0.1$ is the negative interfacial heat flux values shown in the upstream regions for intermediate times. The very high thermal inertia in the wall for very small α_{wf} results greater temperatures in the fluid side than in the wall and therefore heat transfer in the opposite direction. On the other hand, such a high thermal inertia results a drastic decrease in the amount of heat transferred in radial direction through the interface and therefore the time to reach the steady state is much longer for $\alpha_{wf} = 0.1$.

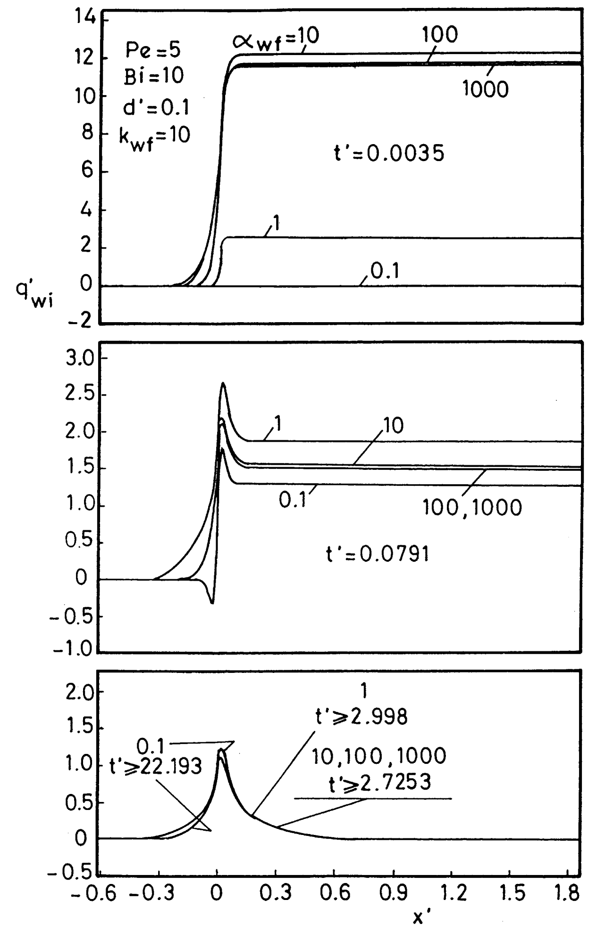

Fig. 5. Effect of wall-to-fluid thermal diffusivity ratio on interfacial heat flux.

Fig. 6 is drawn to see the effect of Peclet number on interfacial heat flux. Since for small Peclet numbers the fluid axial conduction is high, more diffusion of heat back through the upstream region results longer penetration lengths and higher heat flux values in that region. Lower Peclet numbers decrease the degree of convection and therefore the thermal development lengths are greater. The peak values and the rate of change of the heat flux curves are also decrease with decreasing *Pe* number.

In Fig. 7 the interfacial heat flux curves are drawn at several instants of time by parametizing for some Biot numbers. At the initial transient heat flux values are high for increased *Bi*. Since greater *Bi* means more heat transfer to the pipe from the outer surface, the magnitudes of the interfacial heat flux are high for the initial and intermediate periods of time and the rate of thermal development is increased. For the same reason, for high *Bi* numbers the heat flux values are higher in the steady state and the length and the amount of preheating in the upstream region is increased. For very small *Bi* numbers,

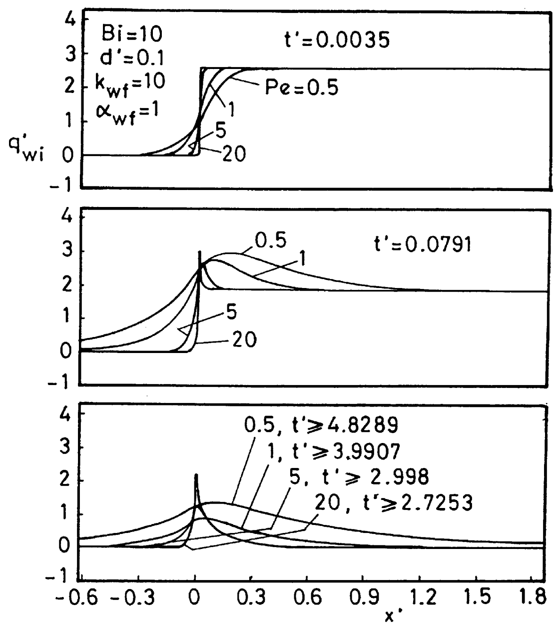

Fig. 6. Effect of Peclet number on interfacial heat flux.

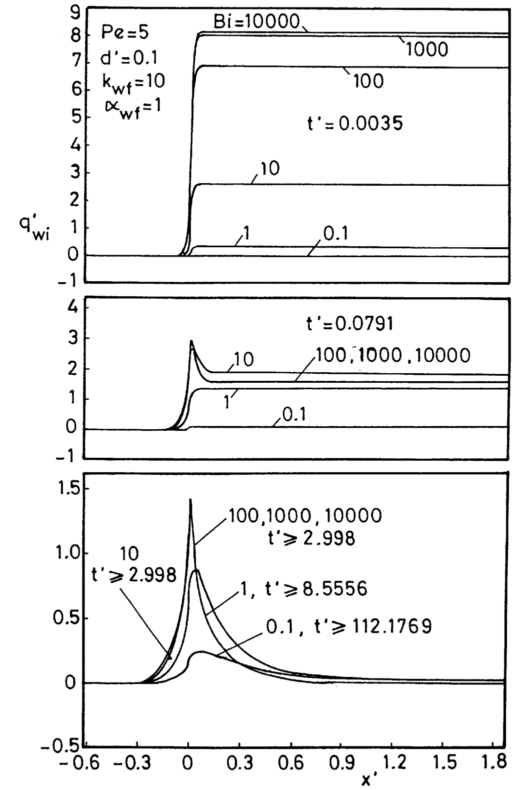

Fig. 7. Effect of Biot number on interfacial heat flux.

the thermal development length becomes considerably large and in fact, the heat flux values do not converge to zero in the fully developed region. Very high convective resistance in the outer wall surface for very small *Bi* numbers probably results for incomplete thermal development. For very small *Bi* numbers the time to reach the steady state is increasing considerably and the peak values and the rate of variation in heat flux curves are decreasing.

5. Conclusions

This work presents an analysis for a transient conjugated heat transfer problem in laminar pipe flow for the thermal entrance region of thick-walled pipes considering the effects of two-dimensional wall and axial fluid conduction. The problem is solved by a numerical finite-difference method for a two-regional pipe for which the upstream region is insulated and the downstream region is faced with an instantaneous step change in the ambient fluid temperature. To understand the effects of five defining parameters, d', k_{wf}, α_{wf}, Pe and Bi, a parametric investigation is done. The results obtained may be outlined as follows.

1. Considerable amount of heat is transferred backward through the upstream side due to the axial conduction both in the wall and in the fluid side and this results preheating of the fluid before entering to the heated downstream region. At the beginning of the transient heat flux values in the upstream region increase first and then decrease. The preheating length is extending more through the upstream region by increasing time. For cases where the backward diffusion of heat is higher in the fluid side than in the wall side, i.e. for very thin walls or very small α_{wf}, reverse heat flow (from the fluid side to the wall) may occur in the upstream region.
2. In the downstream region at early times of transient heat flux values increase rapidly because of high radial conduction in the wall side and attain some constant values in a short distance. As time elapses, with the effect of convection, which is increasing in time and influencing longer distances in the thermal development region, the heat flux values decrease in average and the uniformity of curves is disturbed and after reaching to a maximum value decrease in axial direction. This trend continues until the system reaches to steady state.
3. The effects of wall conjugation and fluid axial conduction on heat transfer characteristics increase with increasing d' and decreasing k_{wf}, α_{wf}, Pe and Bi. The change in computed values for $d' < 0.02$, $k_{wf} > 100$, $\alpha_{wf} > 100$, $Pe > 20$ and $Bi > 100$, a change in each parameter is not significant. The effects of change in parameter values are more sensed in the early periods of transient and decrease as approaching to the steady state.
4. Changes in the parameter values also affect the time to reach the steady state and for each parameter this time increases with increasing effect, i.e. for larger d' and for smaller k_{wf}, α_{wf}, Pe and Bi.

References

[1] Ş. Bilir, Laminar flow heat transfer in pipes including two-dimensional wall and fluid axial conduction, Int. J. Heat Mass Transfer 38 (1995) 1619–1625.
[2] N.E. Wijeysundera, Laminar forced convection in circular and flat ducts with wall axial conduction and external convection, Int. J. Heat Mass Transfer 29 (1986) 797–807.
[3] Ş. Bilir, Numerical solution of Graetz problem with axial conduction, Numer. Heat Transfer Part A 21 (1992) 493–500.
[4] P.J. Schneider, Effects of axial fluid conduction on heat transfer in the entrance region of parallel plates and tubes, Trans. ASME 79 (1957) 765–773.
[5] B. Vick, M.N. Özışık, Effects of axial conduction and convective boundary conditions in slug flow inside a circular tube, Trans. ASME J. Heat Transfer 103 (1981) 436–440.
[6] B. Vick, M.N. Özışık, D.F. Ullrich, Effects of axial conduction in laminar tube flow with convective boundaries, J. Franklin Inst. 316 (1983) 159–173.

[7] S.L. Lee, G.J. Hwang, Finite element solution of low Peclet number fluid flow in round pipe with the Cauchy boundary condition, The Canadian J. Chem. Eng. 59 (1981) 760–765.
[8] A. Campo, J.-C. Auguste, Axial conduction in laminar pipe flows with non-linear wall heat fluxes, Int. J. Heat Mass Transfer 30 (1978) 1597–1607.
[9] S. Olek, Heat transfer in duct flow of non-Newtonian fluids with axial conduction, Int. Commun. Heat Mass Transfer 25 (1998) 928–938.
[10] J. Sucec, Transient heat transfer in the laminar thermal entry region of a pipe: an analytical solution, Appl. Scientific Res. 43 (1986) 115–125.
[11] J. Sucec, An improved quasi-steady approach for transient conjugated forced convection problems, Int. J. Heat Mass Transfer 24 (1981) 1711–1722.
[12] B. Krishan, On conjugated heat transfer in fully developed flow, Int. J. Heat Mass Transfer 25 (1982) 288–289.
[13] J. Sucec, A.M. Sawant, Unsteady conjugated forced convection heat transfer in a parallel plate duct, Int. J. Heat Mass Transfer 27 (1984) 95–101.
[14] J. Sucec, Unsteady conjugated forced convective heat transfer in a duct with convection from the ambient, Int. J. Heat Mass Transfer 30 (1987) 1963–1970.
[15] J. Sucec, Exact solution for unsteady conjugated heat transfer in the thermal entrance region of a duct, Trans. ASME J. Heat Transfer 109 (1987) 295–299.
[16] R.M. Cotta, M.D. Mikhailov, M.N. Özışık, Transient conjugated forced convection in ducts with periodically varying inlet temperature, Int. J. Heat Mass Transfer 30 (1987) 2073–2082.
[17] T.F. Lin, J.C. Kuo, Transient conjugated heat transfer in fully developed laminar pipe flows, Int. J. Heat Mass Transfer 31 (1988) 1093–1102.
[18] W.-M. Yan, Y.L. Tsay, T.F. Lin, Transient conjugated heat transfer in laminar pipe flows, Int. J. Heat Mass Transfer 32 (1989) 775–777.
[19] J.S. Travelho, W.F.N. Santos, Solution for transient conjugated forced convection in the thermal entrance region of a duct with periodically varying inlet temperature, Trans. ASME J. Heat Transfer 113 (1991) 558–562.
[20] S. Olek, E. Elias, E. Wacholder, S. Kaizerman, Unsteady conjugated heat transfer in laminar pipe flow, Int. J. Heat Mass Transfer 34 (1991) 1443–1450.
[21] D.J. Schutte, M.M. Rahman, A. Faghri, Transient conjugate heat transfer in a thick walled pipe with developing laminar flow, Numer.l Heat Transfer Part A 21 (1992) 163–186.
[22] K.-T. Lee, W.-M. Yan, Transient conjugated forced convection heat transfer with fully developed laminar flow in pipes, Numer. Heat Transfer Part A 23 (1993) 341–359.
[23] W.-M. Yan, Transient conjugated heat transfer in channel flows with convection from the ambient, Int. J. Heat Mass Transfer 36 (1993) 1295–1301.
[24] W. Li, S. Kakaç, Unsteady thermal entrance heat transfer in laminar flow with a periodic variation of inlet temperature, Int. J. Heat Mass Transfer 34 (1991) 2581–2592.
[25] Ş. Bilir, Transient conjugated heat transfer in pipes involving two-dimensional wall and axial fluid conduction, Int. J. Heat Mass Transfer 45 (2002) 1781–1788.
[26] J. Sucec, H.Z. Weng, Unsteady conjugated duct heat transfer solution with wall heating, J. Thermophys. Heat Transfer 16 (2002) 128–134.
[27] J. Sucec, Unsteady forced convection with sinusoidal duct wall generation, Int. J. Heat Mass Transfer 45 (2002) 1631–1642.
[28] S.M. Kiwan, M.A. Al-Nimr, Analytical solution for conjugated heat transfer in pipes and ducts, Heat Mass Transfer 45 (2002) 513–516.
[29] W.-M. Yan, Unsteady conjugated heat transfer in turbulent channel flows with convection from the ambient, Int. J. Heat Mass Transfer 38 (1995) 2101–2108.
[30] S.V. Patankar, Numerical Heat Transfer and Fluid Flow, Hemisphere, Washington, DC, 1980.